EE 230
Lecture 25

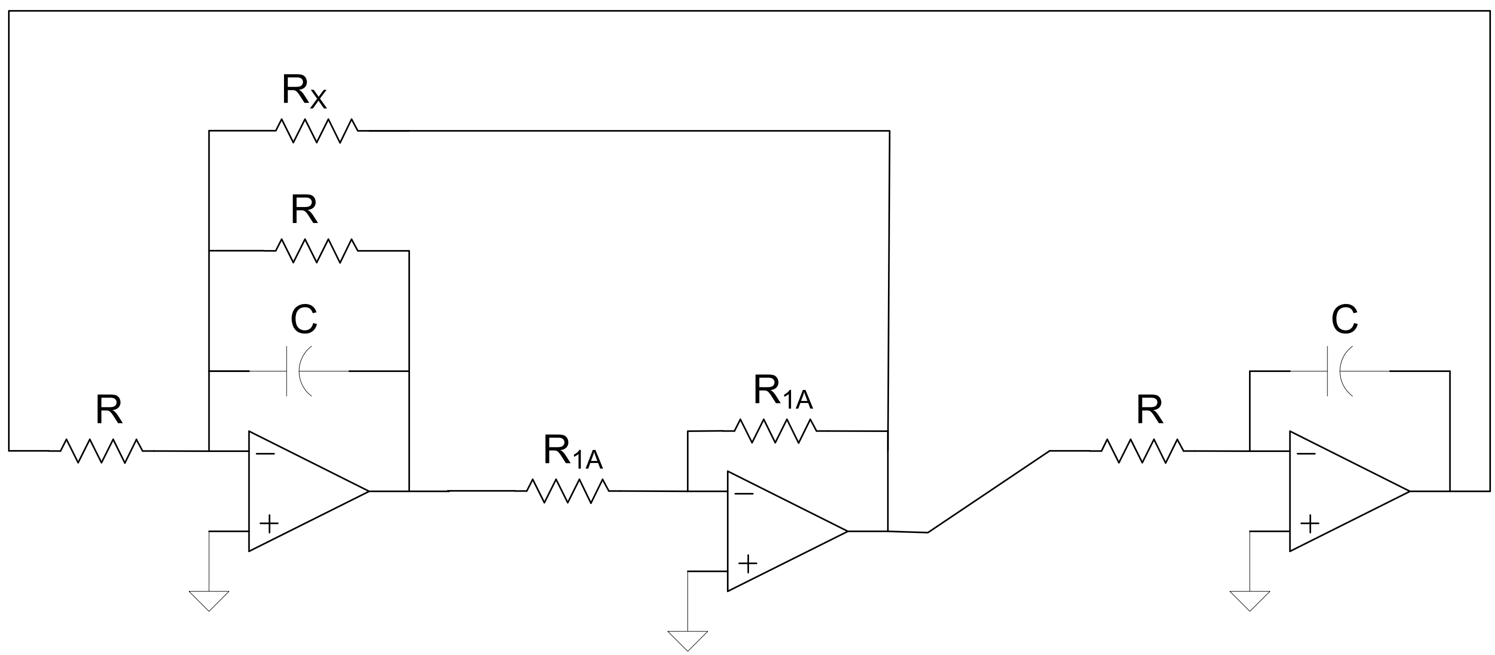
Waveform Generators

- Sinusoidal Oscillators

The Wein-Bridge Structure

Quiz 19

The circuit shown has been proposed as a sinusoidal oscillator. Determine the oscillation criteria and the frequency of oscillation. Assume the op amps are ideal.



And the number is ?

1 3 8

5 4

?

2 6

9 7

Quiz 19

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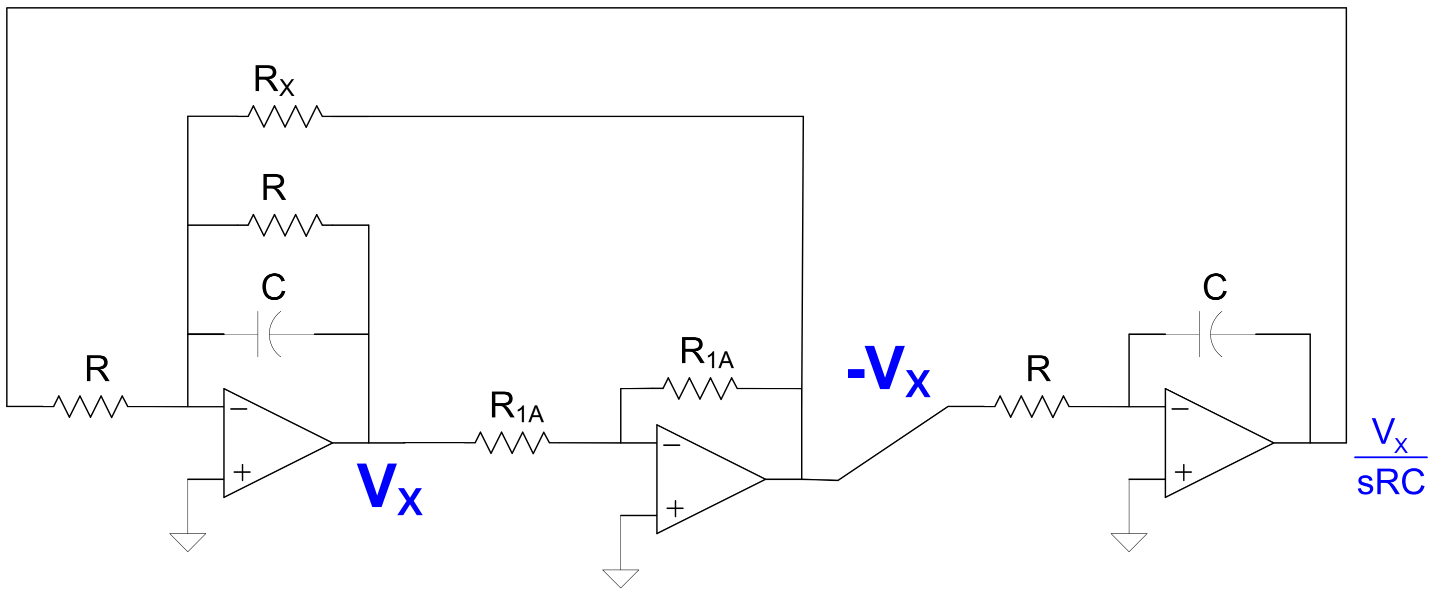


Solution:

$$V_X\left(sC+G-G_X+\frac{G}{sRC}\right)=0$$

$$D(s)=\left(s^2+s\left(\frac{G-G_X}{C}\right)+\frac{1}{(RC)^2}\right)$$

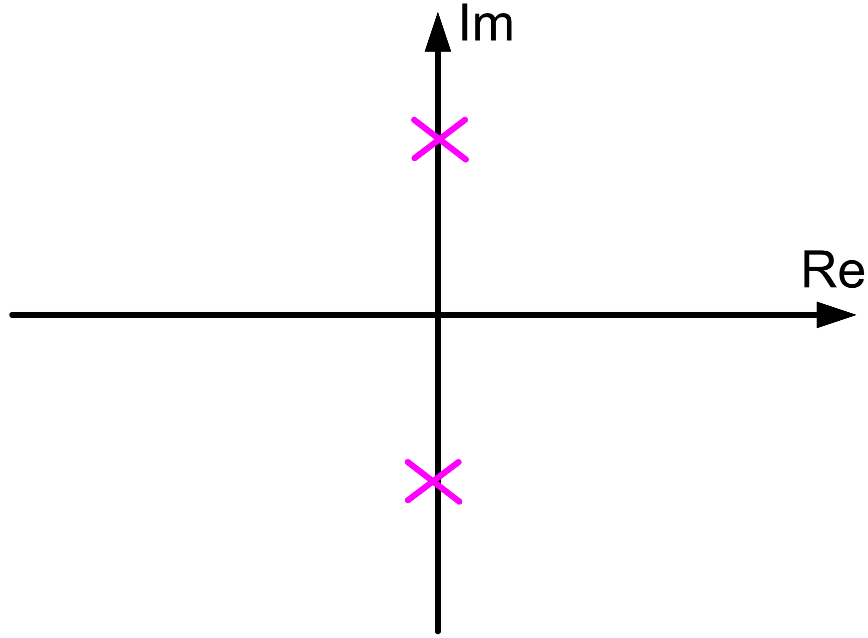
$$D(s)=\left(s^2C^2+sC\left(G-G_X\right)+G^2\right)$$

Oscillation criteria: $G=G_X$

$$\omega_{OSC}=\frac{1}{RC}$$

Review from Last Time:

Sinusoidal Oscillation



A circuit with a single complex conjugate pair of poles on the imaginary axis at +/- jβ will have a sinusoidal output given by

$$X_{OUT}(t)=2\left|\hat{a}_k\right|\sin(\beta t+\theta)$$

The frequency of oscillation will be β rad/sec but the amplitude and phase are indeterminate

Review from Last Time:

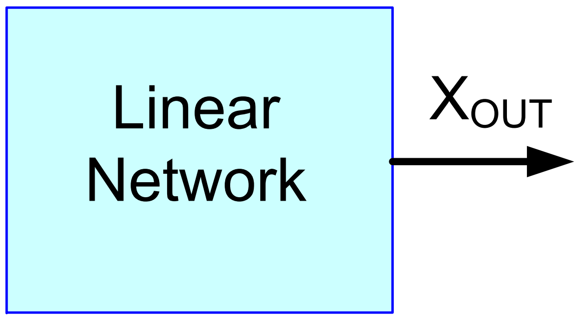
Sinusoidal Oscillation Criteria

A network that has a single complex conjugate pair on the imaginary axis at $\pm j\omega$ and no RHP poles will have a sinusoidal output of the form $X_0(t)=A\sin(\omega t+\theta)$

A and θ can not be determined by properties of the linear network

Review from Last Time:

Characteristic Equation Requirements for Sinusoidal Oscillation



Characteristic Equation Oscillation Criteria:

If the characteristic equation D(s) has exactly one pair of roots on the imaginary axis and no roots in the RHP, the network will have a sinusoidal signal on every nongrounded node.

Review from Last Time:

Relationship between Barkhausen Criteria and Characteristic Equation Criteria for Sinusoidal Oscillation

Characteristic Equation Oscillation Criteria (CEOC)

If the characteristic equation D(s) has exactly one pair of roots on the imaginary axis and no roots in the RHP, the network will have a sinusoidal signal on every nongrounded node

Barkhausen Oscillation Criteria

A feedback amplifier will have sustained oscillation if $A\beta = -1$

Differences:

1. Barkhausen requires a specific feedback amplifier architecture
2. Sustained oscillation says nothing about wave shape

Challenge:

It is impossible to place the poles of any network exactly on the imaginary axis

Sinusoidal Oscillator Design Approach:

Place on pair of cc poles slightly in RHP and have no other RHP poles

With this approach, will observe minor distortion of output waveforms

Review from Last Time:

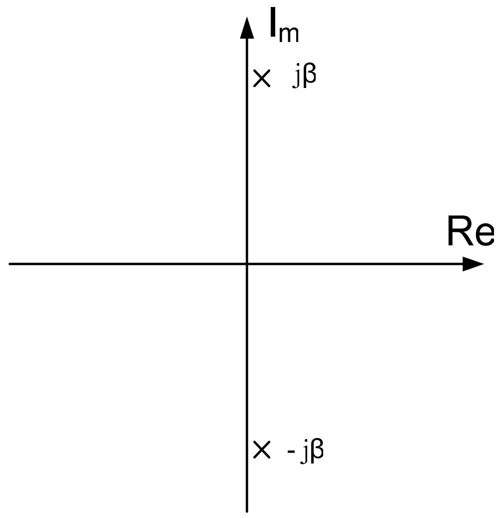
Sinusoidal Oscillator Design Strategy

Build networks with exactly one pair of complex conjugate roots slightly in the RHP and use nonlinearities in the amplifier part of the network to limit the amplitude of the output (i.e $p = \alpha \pm j\beta$ α is very small but positive)

Nonlinearity will cause a small amount of distortion

Frequency of oscillation will be very close to but deviate slightly from β

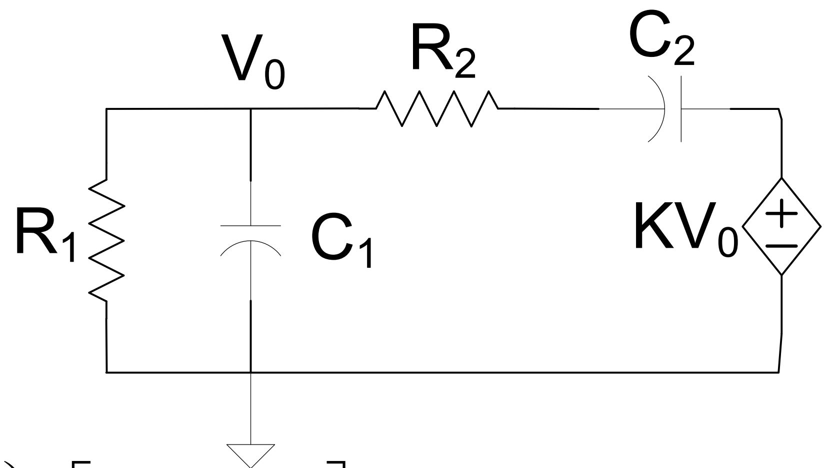
Must be far enough in the RHP so the process and temperature variations do not cause movement back into LHP because if that happened, oscillation would cease!



<u>Review from Last Time:</u>

Sinusoidal Oscillator Design

Consider the following circuit:



$$D(s) = s^2 + s\left(\frac{1}{R_2C_2} + \frac{1}{R_1C_1} + \frac{1-K}{R_2C_1}\right) + \left[\frac{1}{R_1R_2C_1C_2}\right] \qquad \omega_{OSC} = \sqrt{\frac{1}{R_1R_2C_1C_2}}$$

Oscillation Condition:

$$\frac{1}{R_2C_2} + \frac{1}{R_1C_1} + \frac{1-K}{R_2C_1} = 0$$

This is achieved by having K satisfy the equation

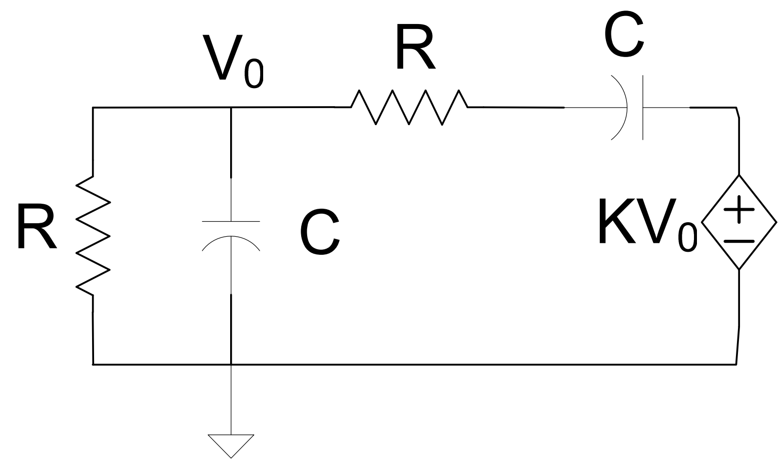
$$K = 1 + \frac{C_1}{C_2} + \frac{R_2}{R_1}$$

Review from Last Time:

Sinusoidal Oscillator Design

Consider the special practical case where $R_1=R_2=R$ and $C_1=C_2=C$:

$$K = 1 + \frac{C_1}{C_2} + \frac{R_2}{R_1} = 3$$



$$\omega_{OSC} = \frac{1}{RC}$$

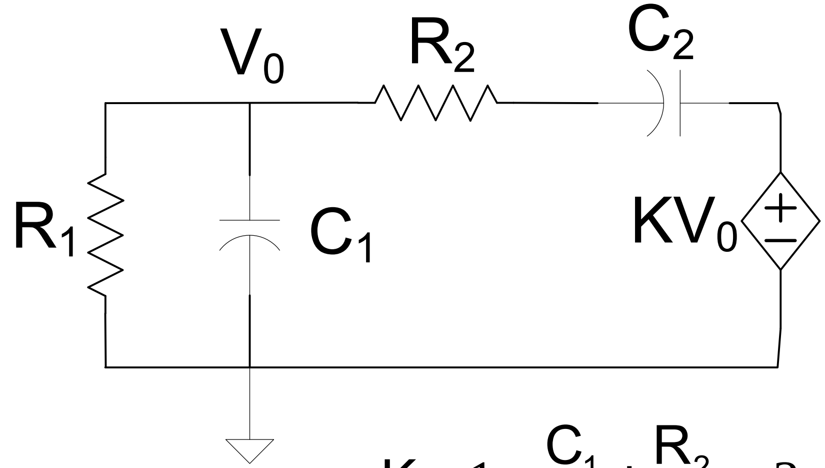
This is termed the Wein Bridge Oscillator

One of the most popular sinusoidal oscillator structures

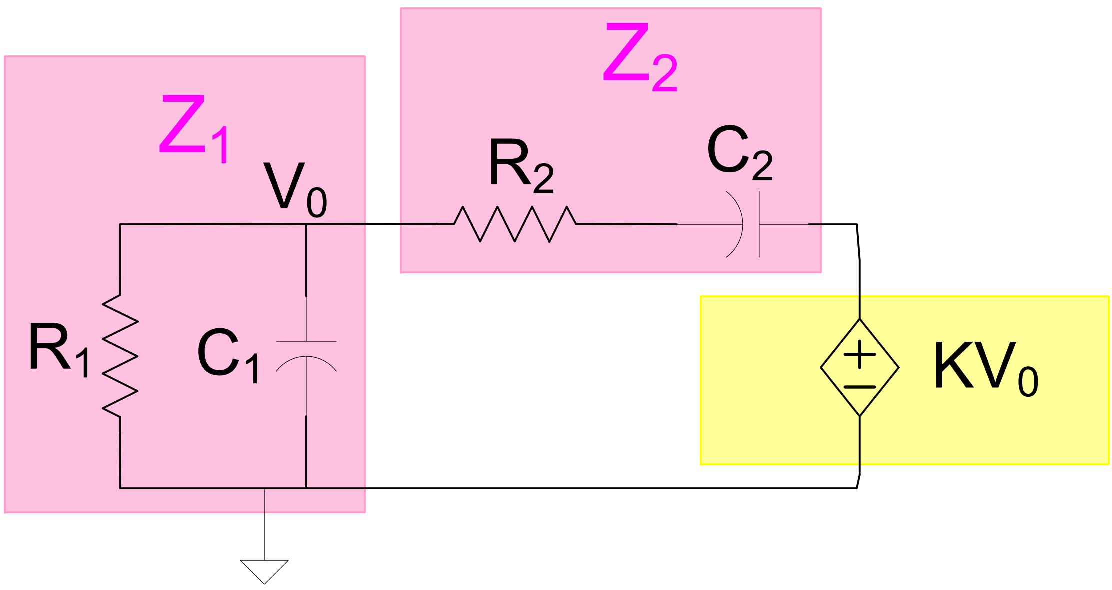
Practically make K slightly larger than 3 and judiciously manage the nonlinearities to obtain low distortion

The Wein-Bridge Oscillator

Another Perspective:

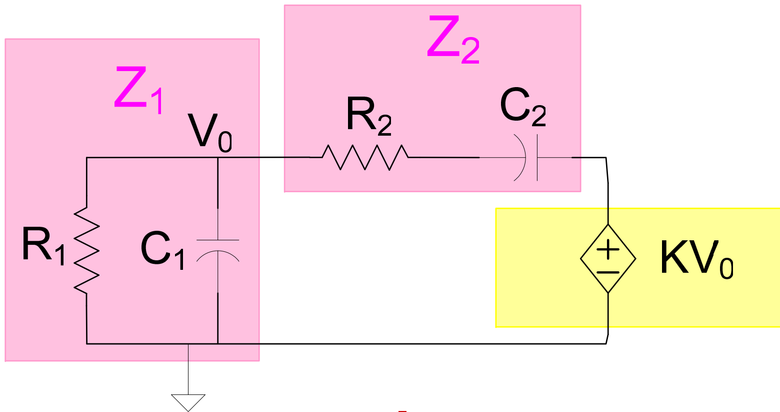


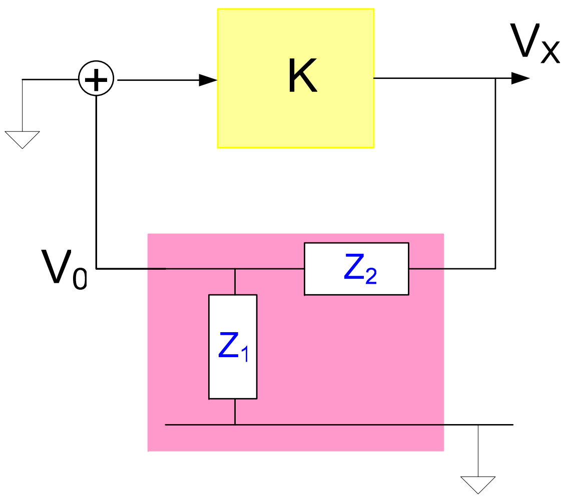
$$K = 1 + \frac{C_1}{C_2} + \frac{R_2}{R_1} = 3$$

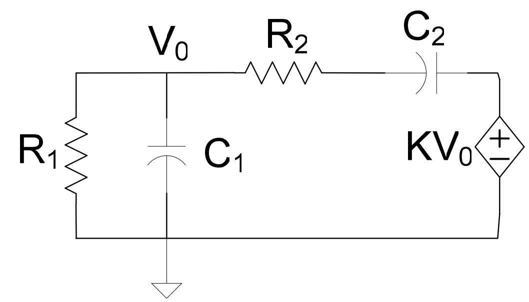


The Wein-Bridge Oscillator

Another Perspective:







$$K = 1 + \frac{C_1}{C_2} + \frac{R_2}{R_1} = 3$$

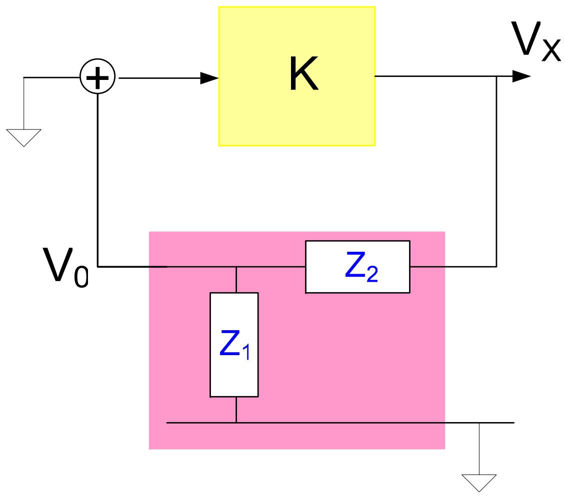
Note this is a feedback amplifier with gain K and

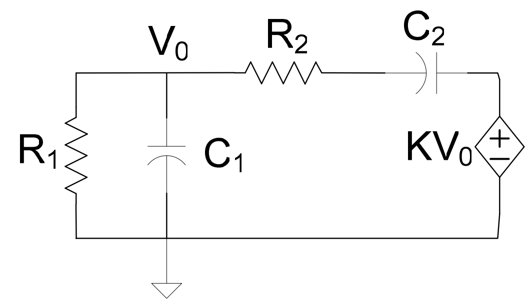
$$\beta = \frac{Z_1}{Z_1 + Z_2}$$

Lets check Barkhausen Criteria for this circuit

The Wein-Bridge Oscillator

Lets check Barkhausen Criteria for this circuit





$$\beta = \frac{Z_1}{Z_1+Z_2}$$

$$K = 1 + \frac{C_1}{C_2} + \frac{R_2}{R_1} = 3$$

$$K\beta = \frac{Z_1}{Z_1+Z_2} = \frac{KR_1C_1s}{s^2R_1R_2C_1C_2+s\left(R_1C_2+R_1C_1+R_2C_2\right)+1}$$

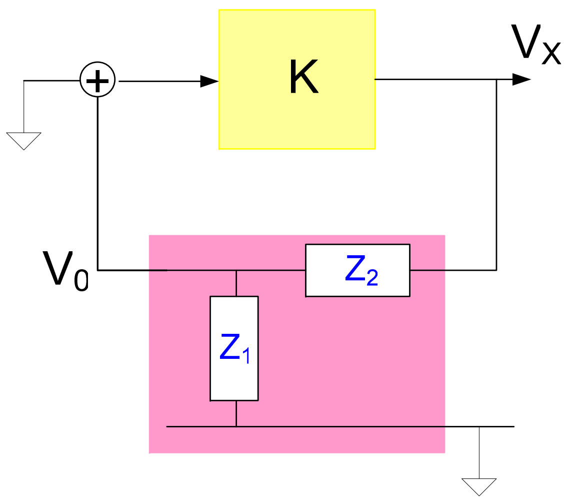
Setting Kβ= -1, obtain

$$\frac{KR_1C_1s}{s^2R_1R_2C_1C_2+s\left(R_1C_2+R_1C_1+R_2C_2\right)+1} = -1$$

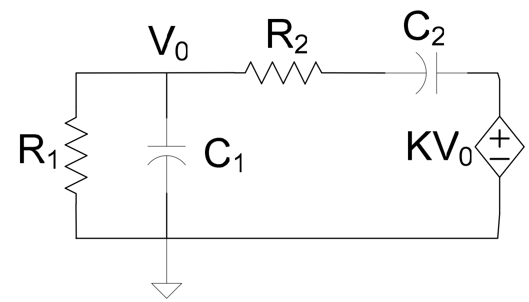
$$s^2R_1R_2C_1C_2+s\left(R_1C_2+R_1C_1+R_2C_2-KR_1C_1\right)+1=0$$

The Wein-Bridge Oscillator

Lets check Barkhausen Criteria for this circuit



$$\beta = \frac{Z_1}{Z_1 + Z_2}$$



$$K = 1 + \frac{C_1}{C_2} + \frac{R_2}{R_1} = 3$$

Putting in s= jω, obtain the Barkhausen criteria

$$\left[1-\omega^2 R_1 R_2 C_1 C_2\right] + j\left[\omega\left(R_1 C_2 + R_1 C_1 + R_2 C_2 - K R_1 C_1\right)\right] = 0$$

Solving, must have (from the imaginary part)

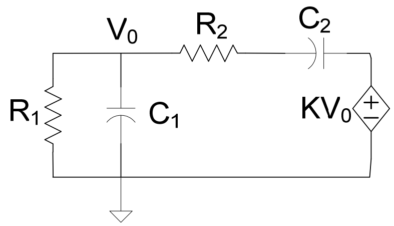
$$K = 1 + \frac{C_1}{C_2} + \frac{R_2}{R_1}$$

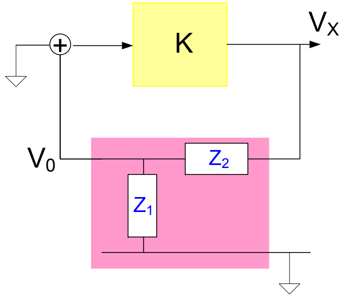
And this will occur at the oscillation frequency of (from real part)

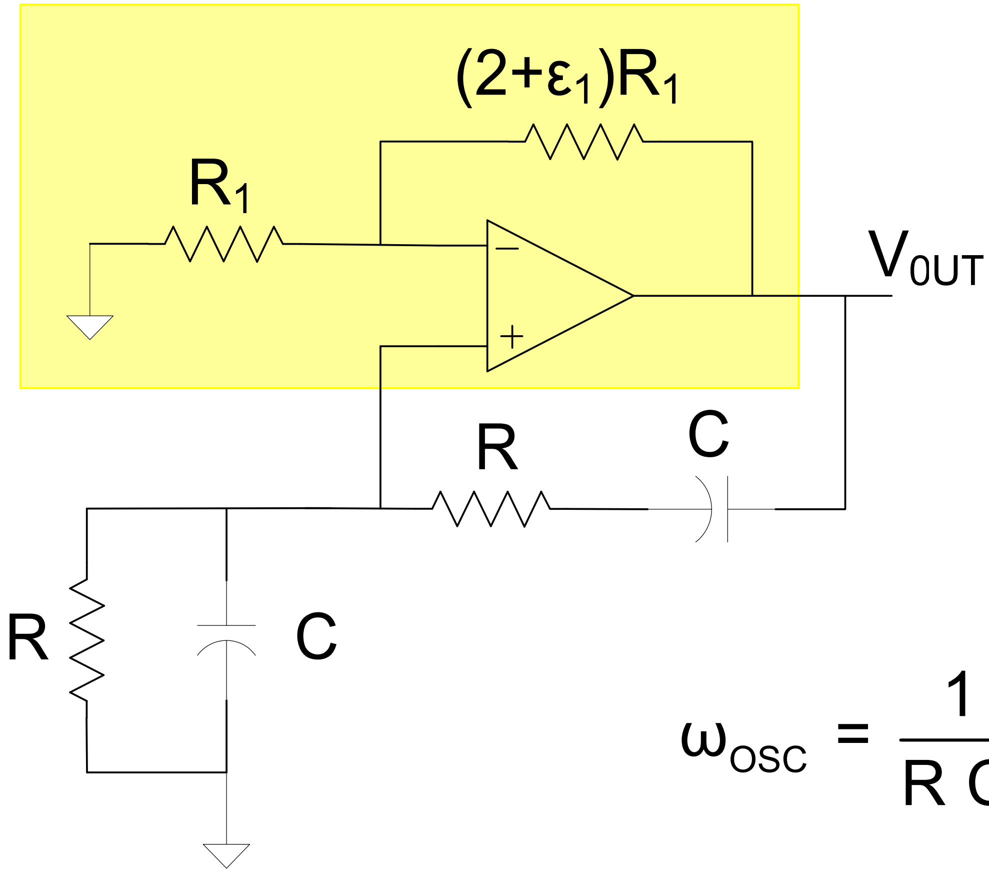
$$\omega_{OSC} = \sqrt{\frac{1}{R_1 R_2 C_1 C_2}}$$

The Wein-Bridge Oscillator

Basic implementation for the equal R, equal C case

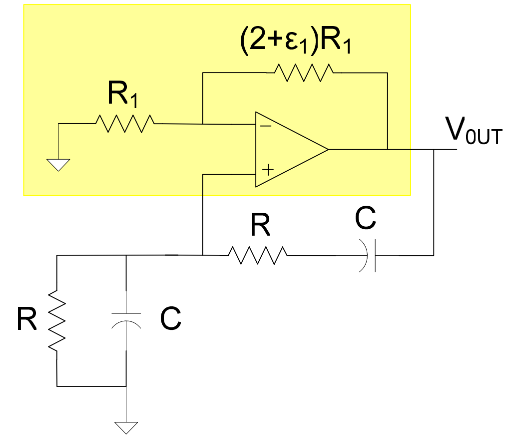




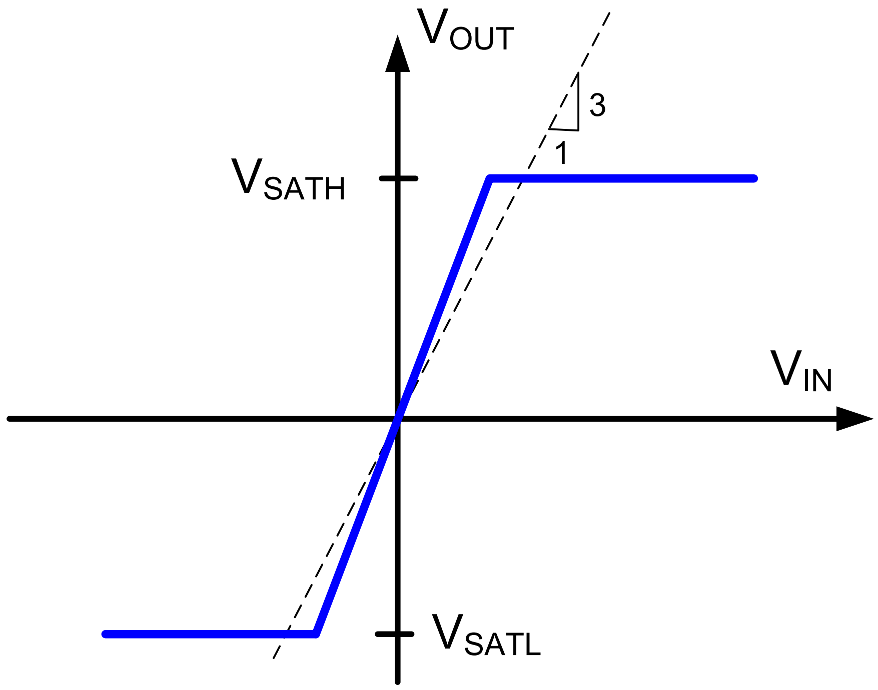


$$\omega_{OSC} = \frac{1}{R\,C}$$

The Wein-Bridge Oscillator



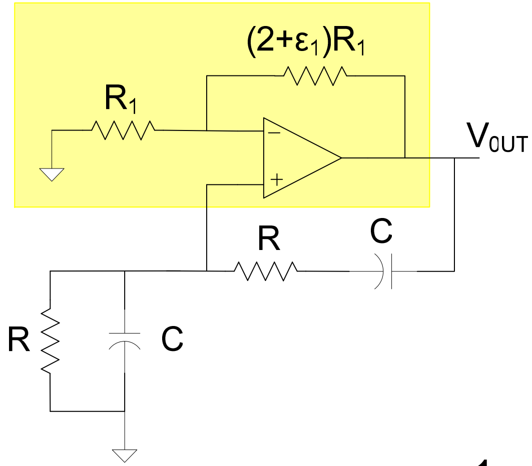
Amplifier Transfer Characteristics



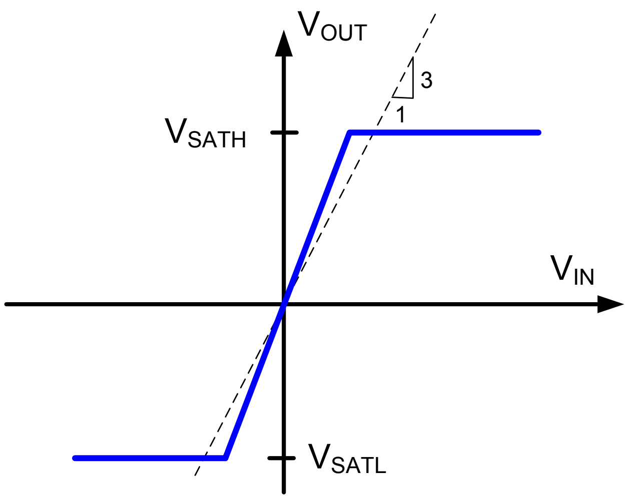
$$\omega_{OSC} = \frac{1}{RC}$$

Slope slightly larger than 3
Amplitude of oscillation will be approximately V_{SATH} (assuming V_{SATH}=-V_{STATL})
Distortion introduced by the abrupt nonlinearities when clipping occurs

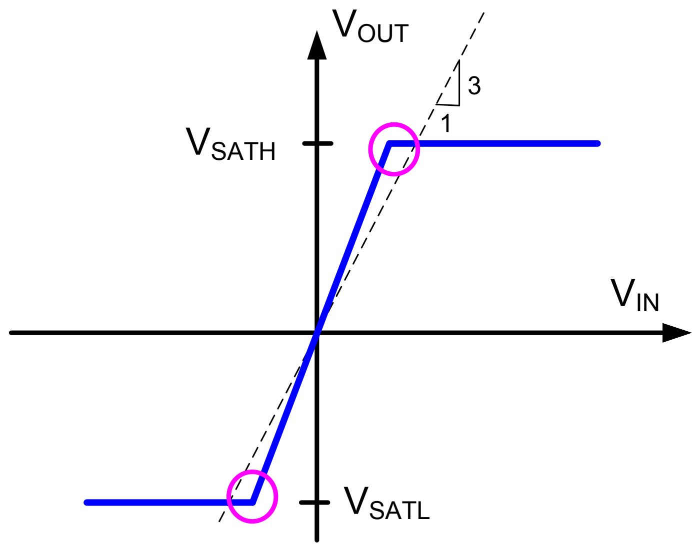
The Wein-Bridge Oscillator



Amplifier Transfer Characteristics



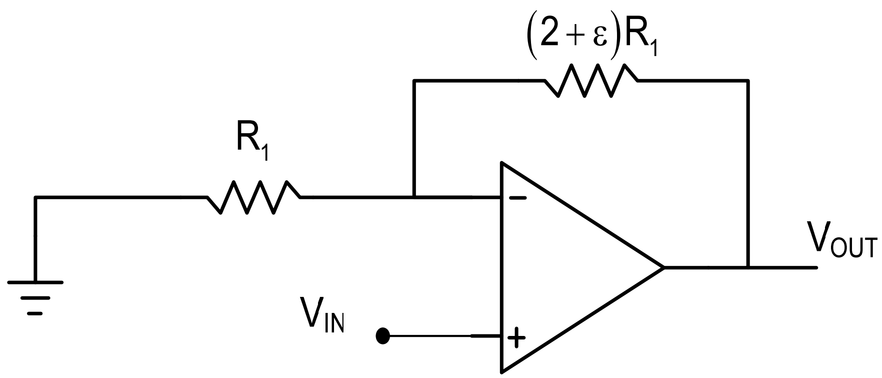
$$\omega_{OSC} = \frac{1}{R\,C}$$

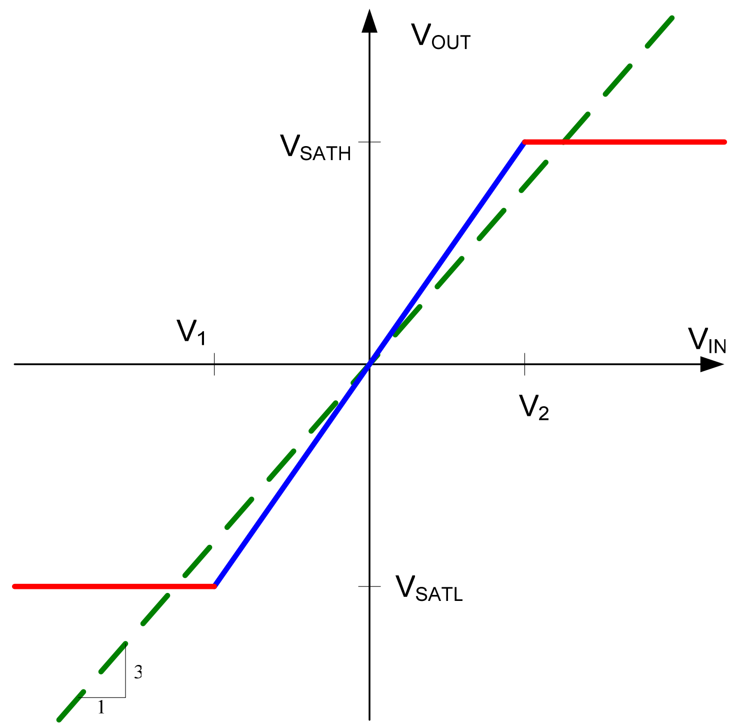


Abrupt nonlinearities cause distortion

Better performance (reduced nonlinearity) can be obtained by introducing less abrupt nonlinearities to limit amplitude

Amplitude Limiting in Noninverting Amplifier Structure





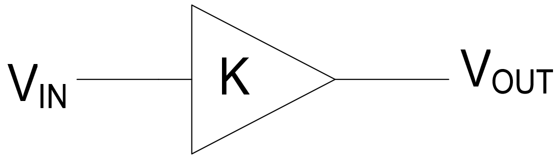
Observe:

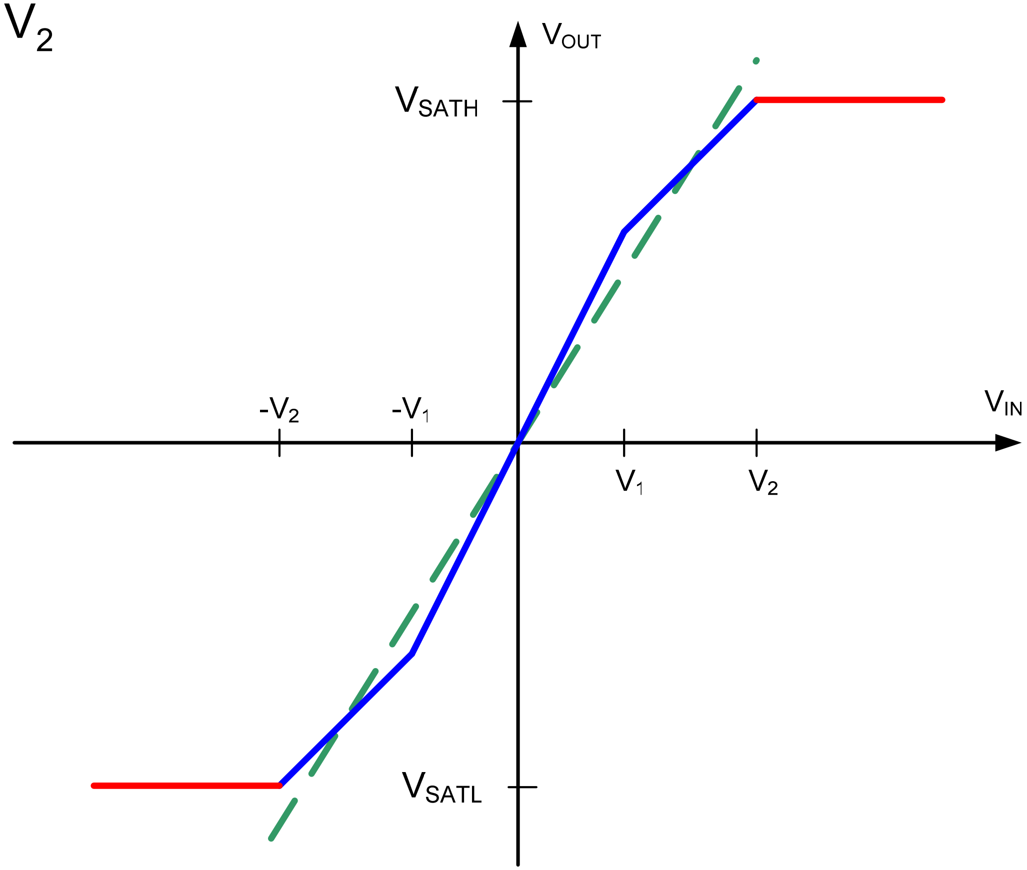
- Amplifier gain changes from 3 + ε for $V_1 < V_{IN} < V_2$ to 0 for $V_{IN}<V_1$ or $V_{IN}>V_2$
- V_{SATH} and V_{SATL} strongly dependent upon op amp bias voltages V_{DD} and V_{SS}
- This nonlinearity in the real amplifier will limit the output signal amplitude
- Can cause rather significant distortion

Can we do this?

Obtain slope >3 for $-V_1 < V_{IN} < V_1$ and slope <3 for $V_1 < V_{IN} < V_2$ and for $-V_2 < V_{IN} < -V_1$

Limit V_{IN} to interval $-V_2 < V_{IN} < V_2$

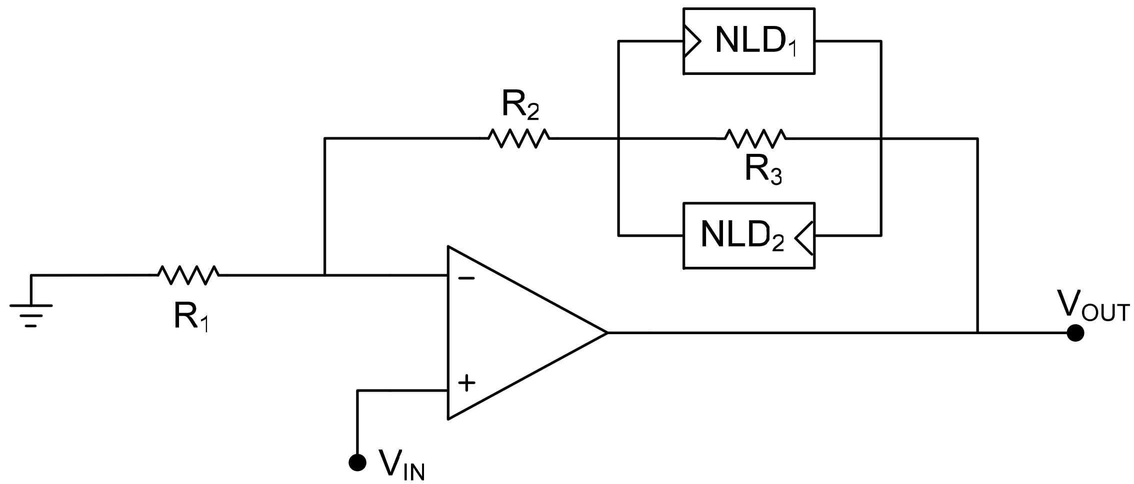


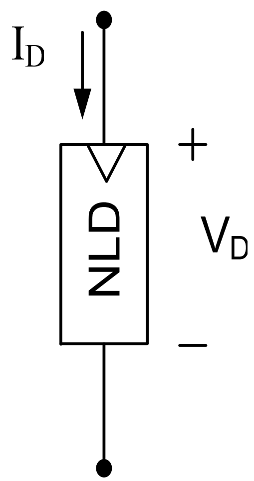


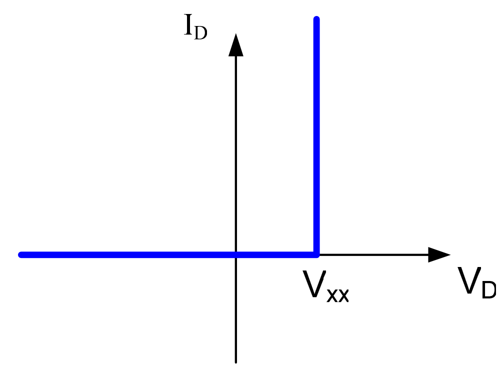
– If possible, hard nonlinearity associated with amplifier saturation will not be excited

– Dramatic reduction in distortion is anticipated

Consider:







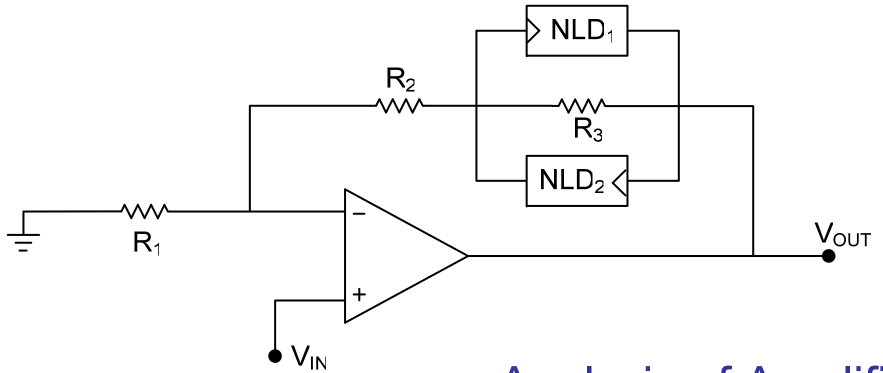
NLD Transfer Characteristics

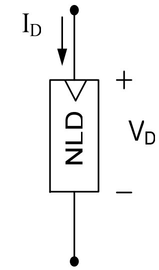
$V_D = V_{XX}$ for $I_D > 0$

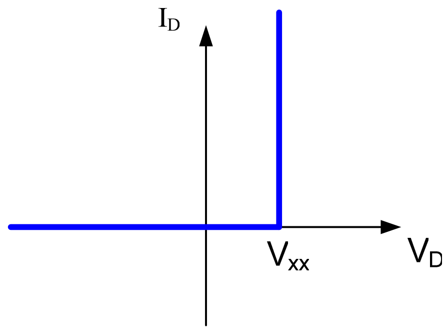
$I_D = 0$ for $V_D < V_{XX}$

(will assume $V_{XX} > 0$)

Consider:





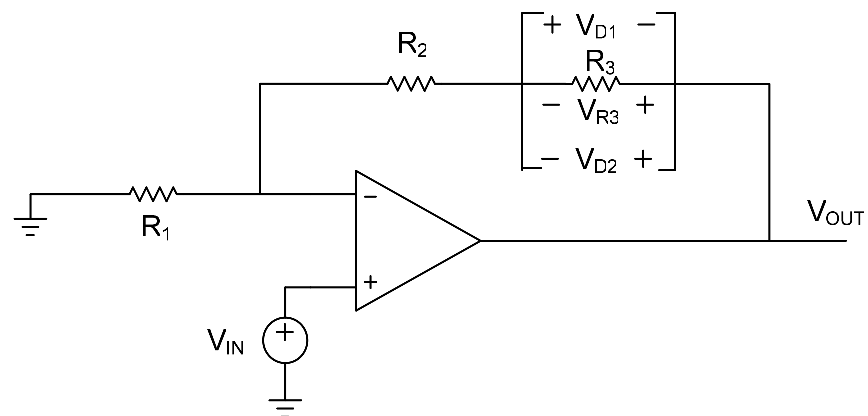


$V_D = V_{XX}$ for $I_D > 0$

$I_D = 0$ for $V_D < V_{XX}$

Analysis of Amplifier Circuit:

Case 1: $I_{D1} = 0$ and $I_{D2} = 0$



Solution:

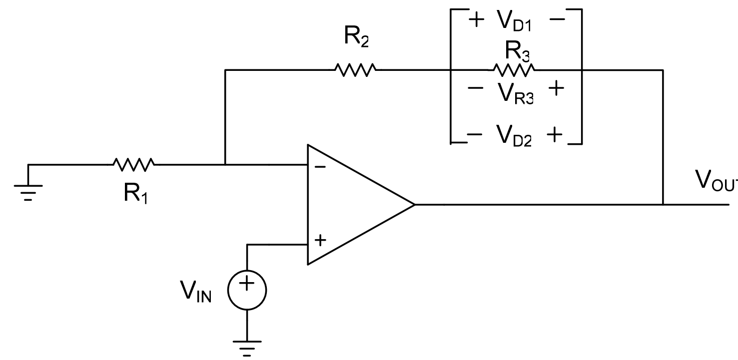
$$V_{OUT} = \left[1 + \frac{(R_2 + R_3)}{R_1}\right] V_{IN}$$

Must determine where this part of the solution is valid

Valid for $V_{D1} < V_{XX}$ and $V_{D2} < V_{XX}$

but $V_{D1} = -V_{R3}$ and $V_{D2} = V_{R3}$

thus, valid for $V_{R3} > -V_{XX}$ and $V_{R3} < V_{XX}$



but
$$V_{R3} = \frac{R_3}{R_2 + R_3}\left(V_o - V_{in}\right)$$

$$V_{R3} = \frac{R_3}{R_2 + R_3}\left[1 + \frac{R_2 + R_3}{R_1} - 1\right]V_{in}$$

$$V_{R3} = \frac{R_3}{R_2 + R_3}\left[\frac{R_2 + R_3}{R_1}\right]V_{in}$$

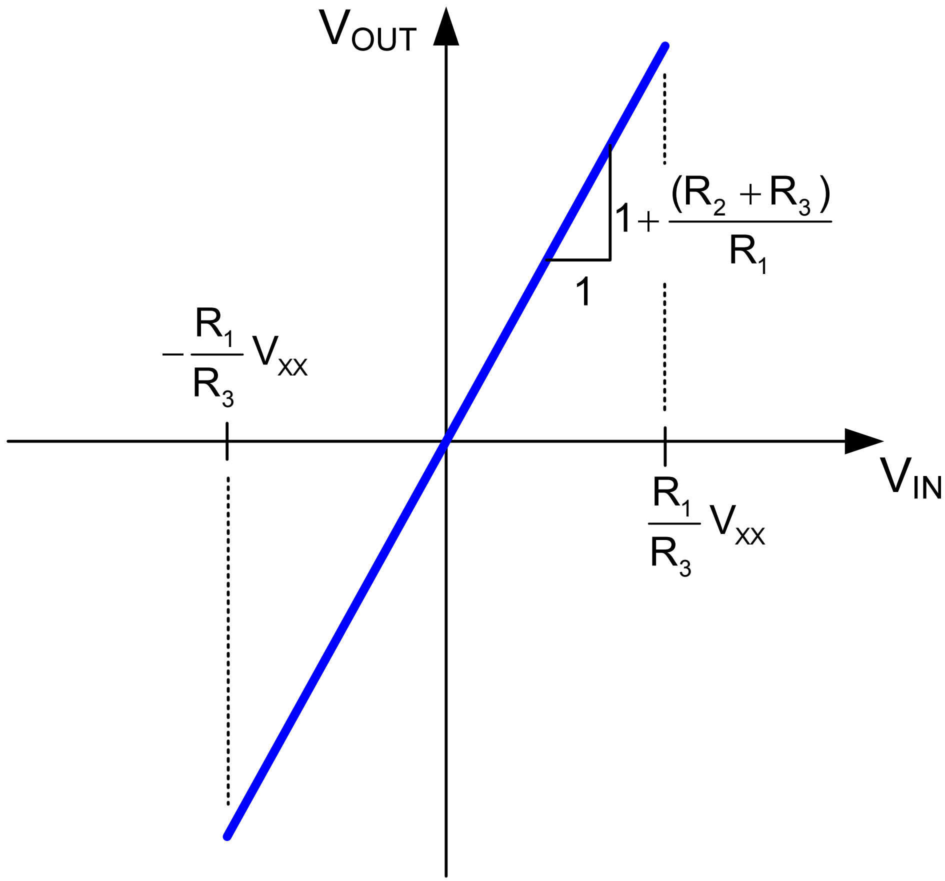
$$V_{R3} = \frac{R_3}{R_1}V_{in}$$

$\therefore$ valid for

$$\frac{R_3}{R_1}V_{IN} < V_{XX} \quad \text{and} \quad \frac{R_3}{R_1}V_{IN} > -V_{XX}$$

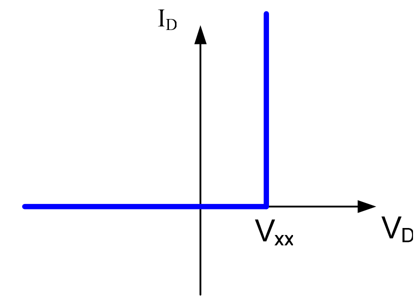
$$-\frac{R_1}{R_3}V_{XX} < V_{IN} < \frac{R_1}{R_3}V_{XX}$$

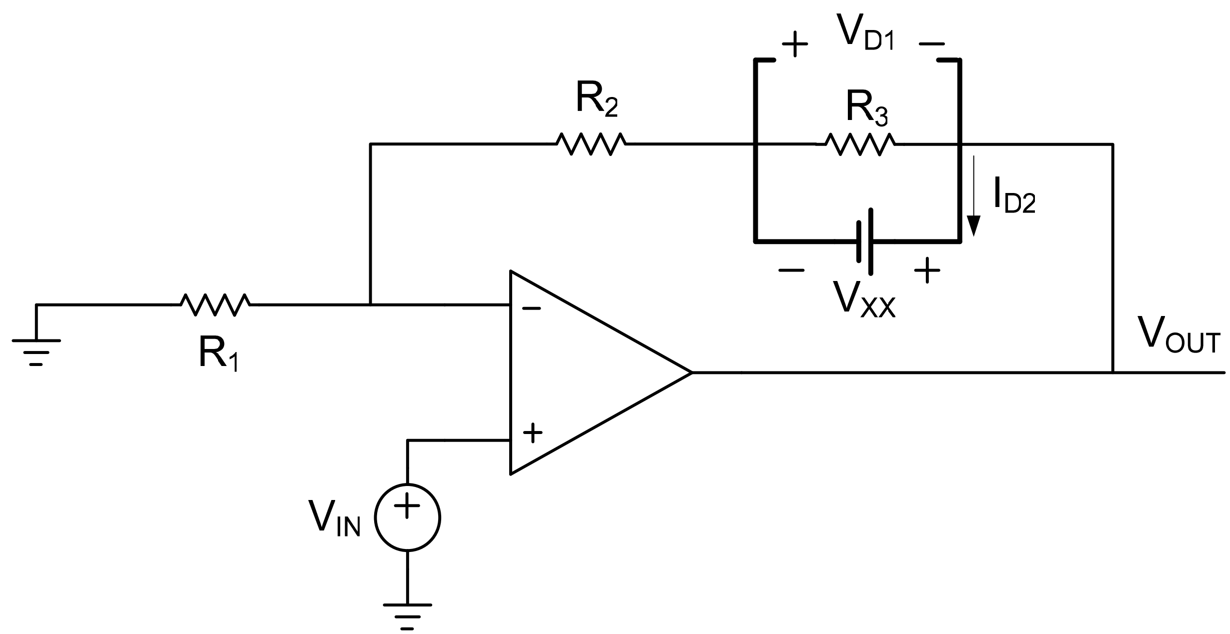
Graph of solution for Case 1



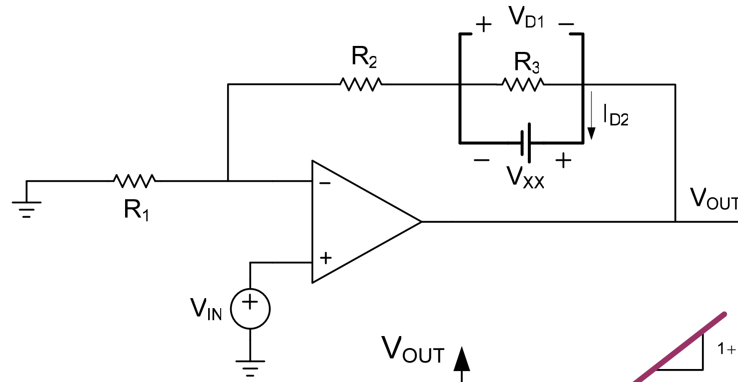
Case 2: NLD$_2$ is in the conducting state ($V_{D2} = V_{XX}$)

NLD$_1$ is in the nonconducting state ($I_{D1} = 0$)



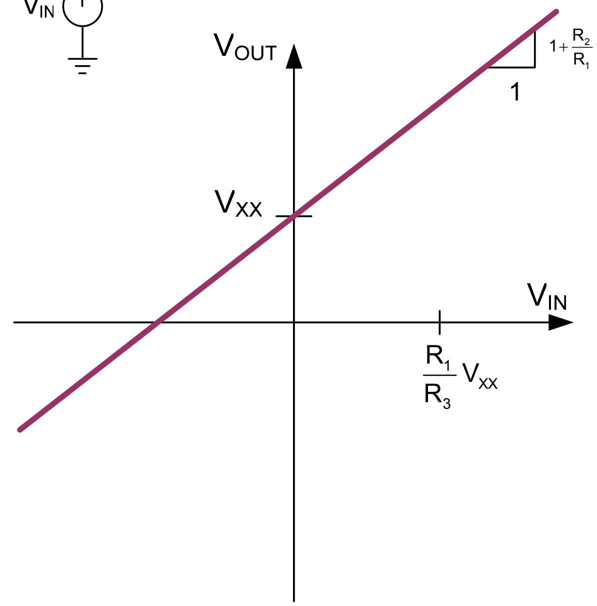


Solution for Case 2 Continued



Applying superposition we obtain

$$V_{OUT} = V_{IN}\left(1+\frac{R_2}{R_1}\right)+V_{xx}$$



This solution is valid for $V_{D1} < 0$ and $I_{D2} > 0$

But: $V_{D1} = -V_{XX}$ and $I_{D2} = \frac{V_{OUT}-V_{IN}-V_{XX}}{R_2} - \frac{V_{XX}}{R_3}$

Substituting the validity conditions, we obtain

$-V_{XX} < 0$ and $\frac{V_{OUT}-V_{IN}-V_{XX}}{R_2} - \frac{V_{XX}}{R_3} > 0$

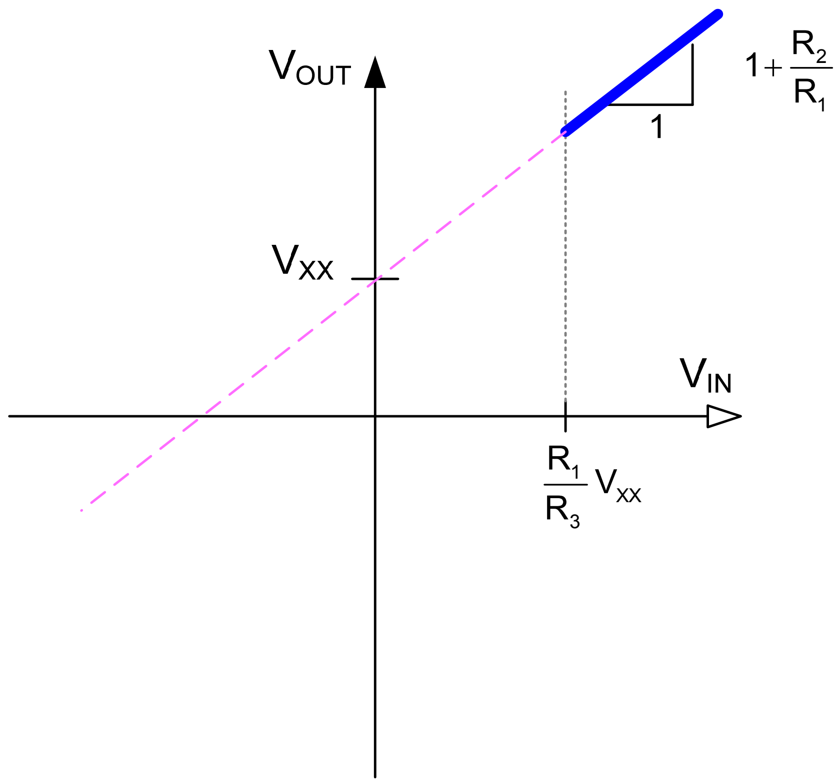
The first of these inequalities is valid provided $V_{XX}>0$ and substituting the expression for V_{OUT} into the second, we obtain after simplification

$$V_{IN} > \frac{R_1}{R_3}V_{XX}$$

Solution for Case 2 Continued

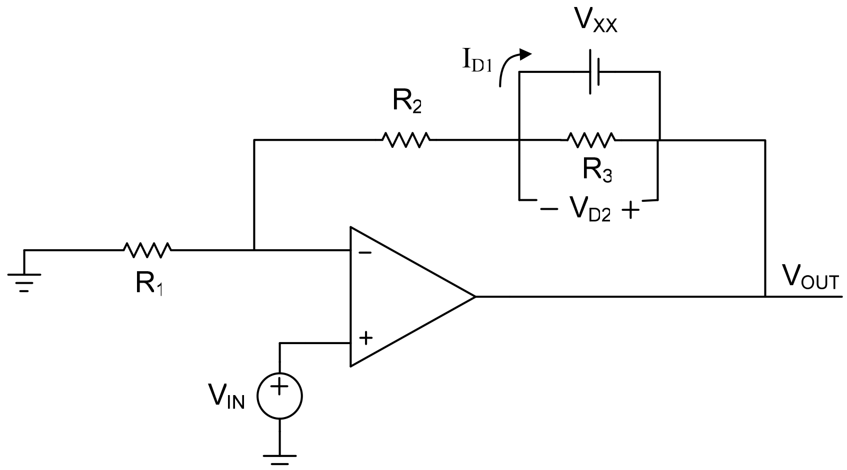
$V_{XX}>0$ $\qquad$ $V_{IN} > \frac{R_1}{R_3} V_{XX}$

Assuming $V_{XX}>0$, the region where Case 2 is valid is thus determined by the second inequality

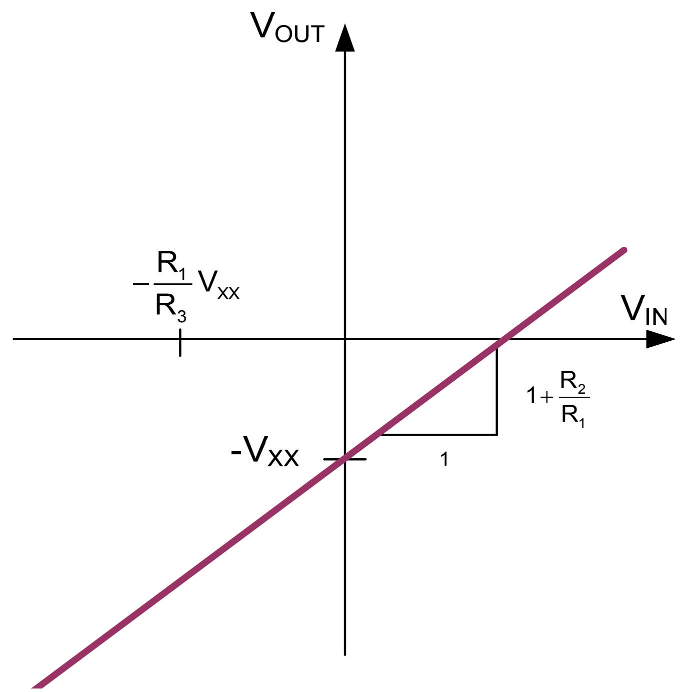


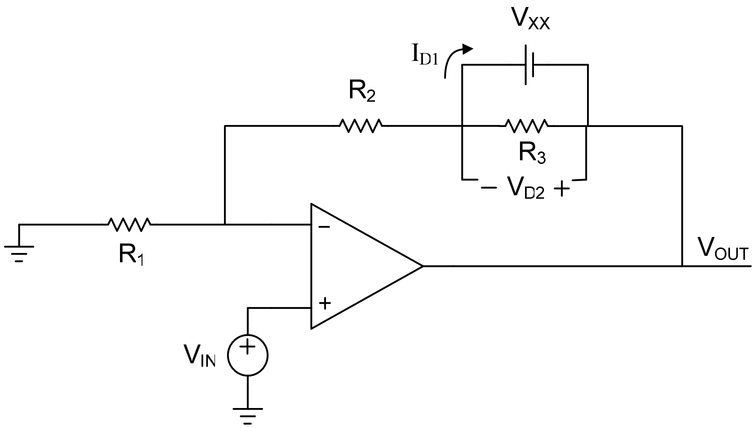
Case 3: NLD$_2$ is nonconducting ($I_{D2} = 0$)

NLD$_1$ is conducting ($V_{D1} = V_{XX}$)



$$V_{OUT} = \left(1 + \frac{R_2}{R_1}\right) V_{IN} - V_{XX}$$





Solution for Case 3 continued:

This solution is valid for $V_{D2} < 0$ and $I_{D1} > 0$

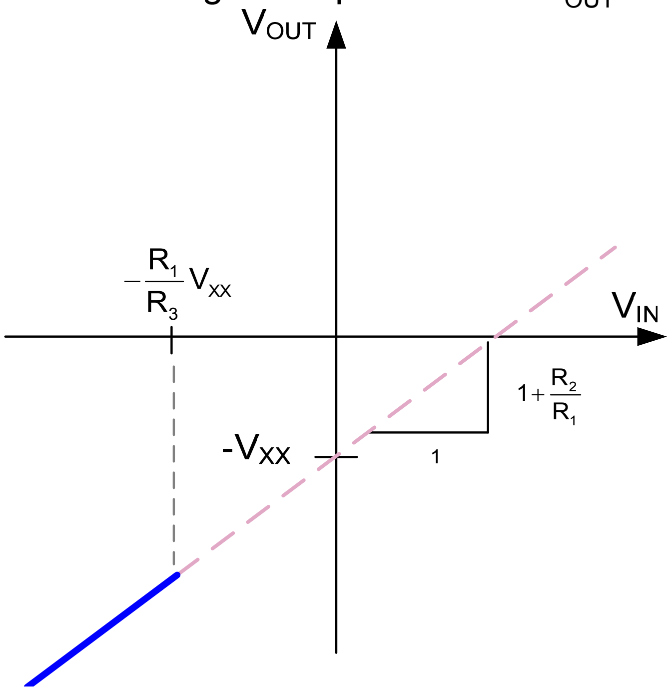
But: $V_{D2} = -V_{XX}$ and $I_{D1} = \dfrac{V_{IN} - V_{OUT} - V_{XX}}{R_2} - \dfrac{V_{XX}}{R_3}$

Substituting the validity conditions, we obtain

$-V_{XX} < 0$ and $\dfrac{V_{IN} - V_{OUT} - V_{XX}}{R_2} - \dfrac{V_{XX}}{R_3} > 0$

The first of these inequalities is valid provided $V_{XX} > 0$ and substituting the expression for V_{OUT} into the second, we obtain after simplification

$$V_{IN} < -\frac{R_1}{R_3} V_{XX}$$



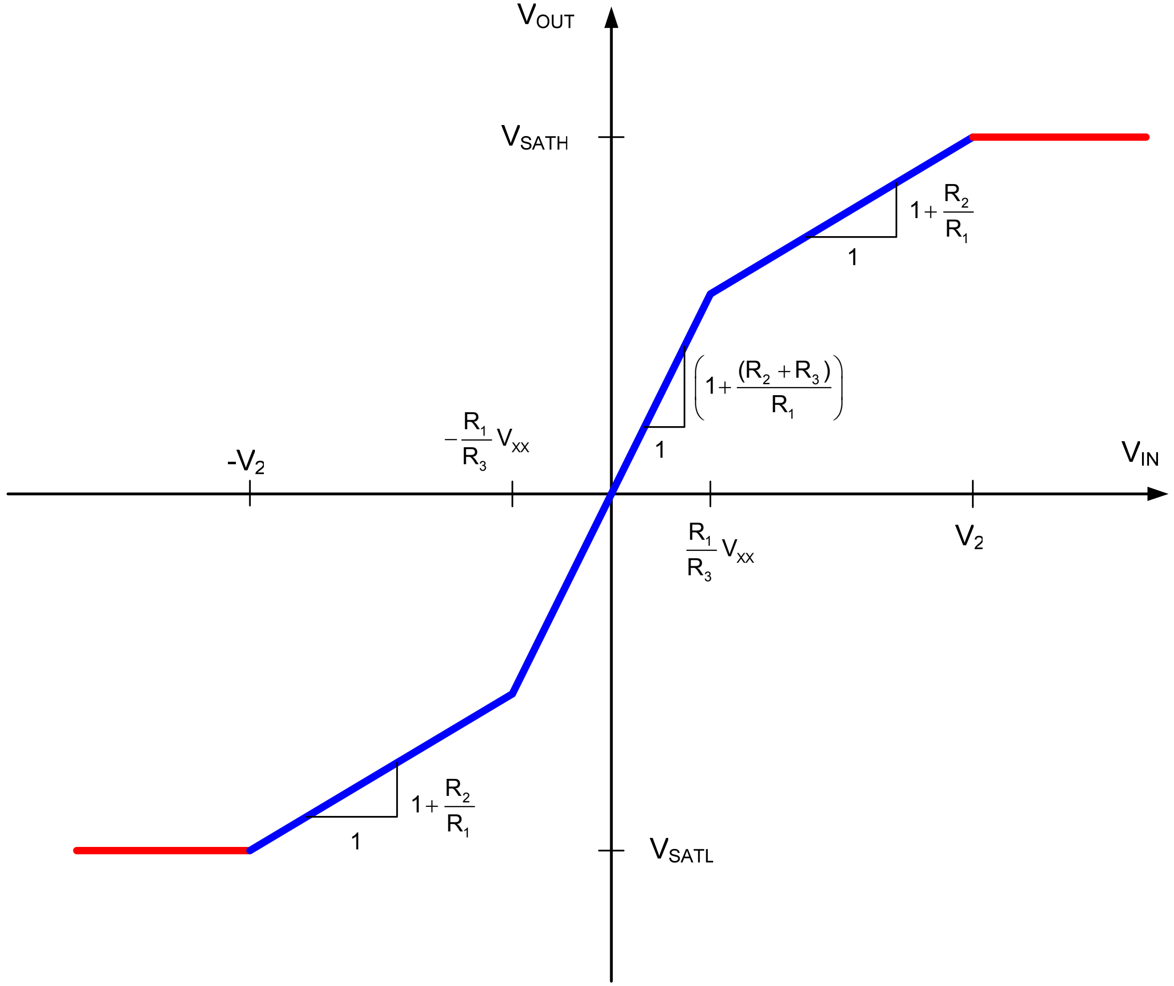
Case 4: NLD$_1$ and NLD$_2$ both conducting (this case never happens and need not be considered since we already have a solution for all inputs)

Thus, if we neglect the saturation of the op amp, we can write an expression for the output as

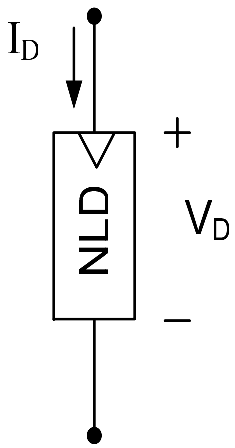
$$V_{OUT} = \begin{cases} \left(1+\dfrac{R_2}{R_1}\right)V_{IN} + V_{xx} & V_{IN} > \dfrac{R_1}{R_3}V_{XX} \quad "2" \\ \left(1+\dfrac{R_2+R_3}{R_1}\right)V_{IN} & -\dfrac{R_1}{R_3}V_{XX} < V_{IN} < \dfrac{R_1}{R_3}V_{XX} \quad "1" \\ \left(1+\dfrac{R_2}{R_1}\right)V_{IN} - V_{xx} & V_{IN} < -\dfrac{R_1}{R_3}V_{XX} \quad "3" \end{cases}$$

This is shown graphically, along with the saturation of the op amp, on the following slide

Overall Transfer Characteristics

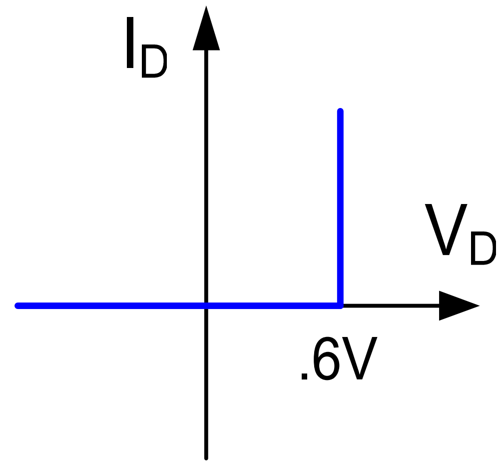


Overall Transfer Characteristics

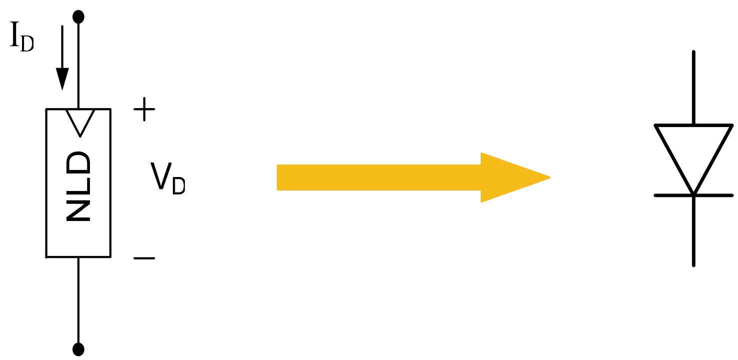


$V_D = V_{XX}$ for $I_D > 0$

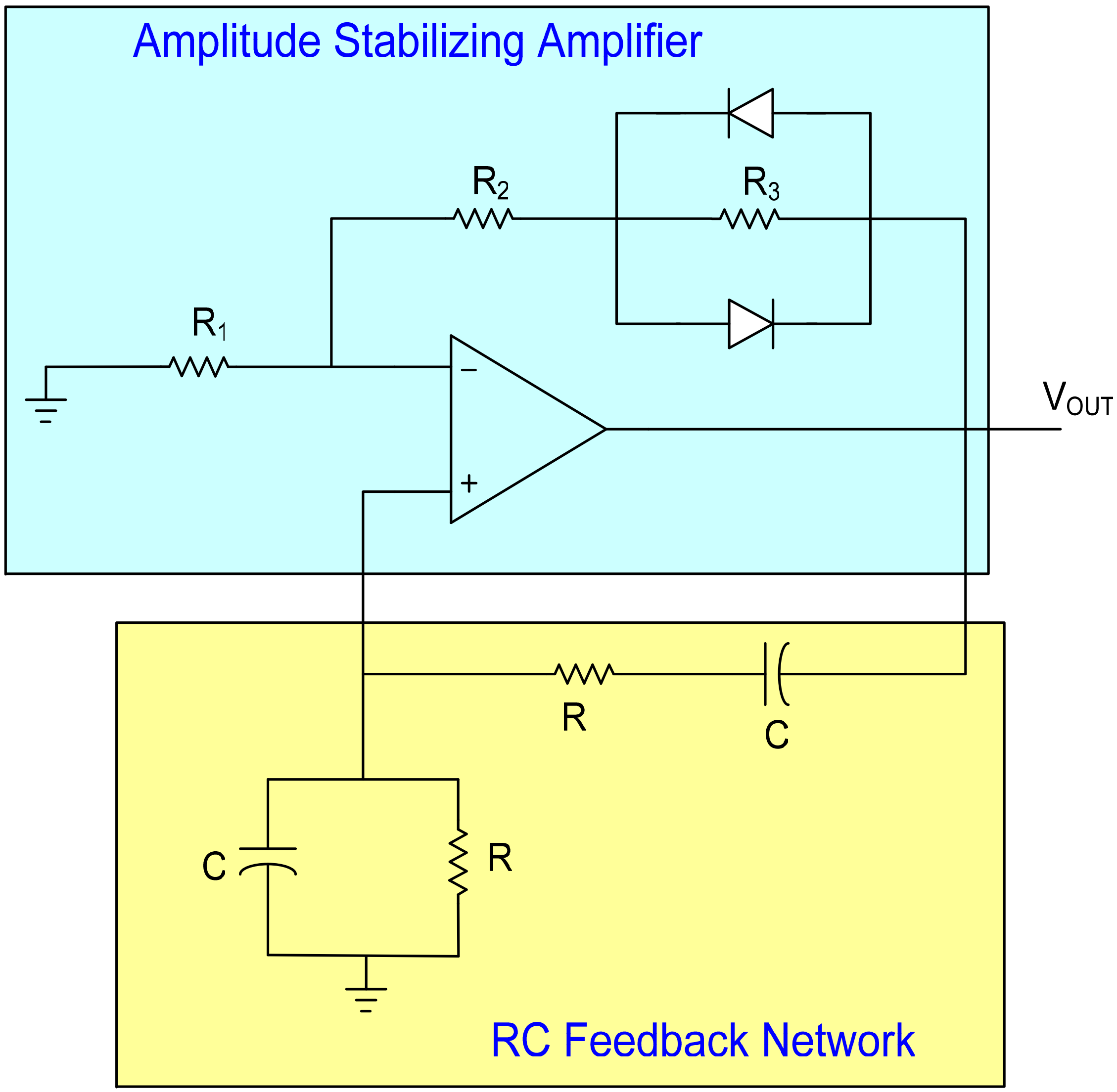
$I_D = 0$ for $V_D < V_{XX}$



If V_{XX}=0.6V, this represents a good approximation to the transfer characteristics of a silicon diode. We thus can replace the NLD with a diode and obtain the amplitude stabilized Wien-Bridge oscillator



Wein – Bridge Oscillator with Amplitude Stabilization



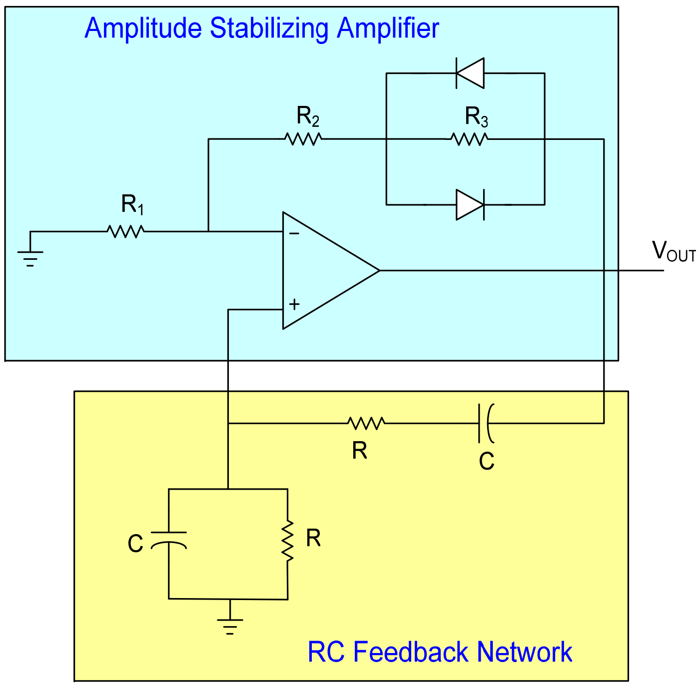
$$R_2 < 2R_1$$

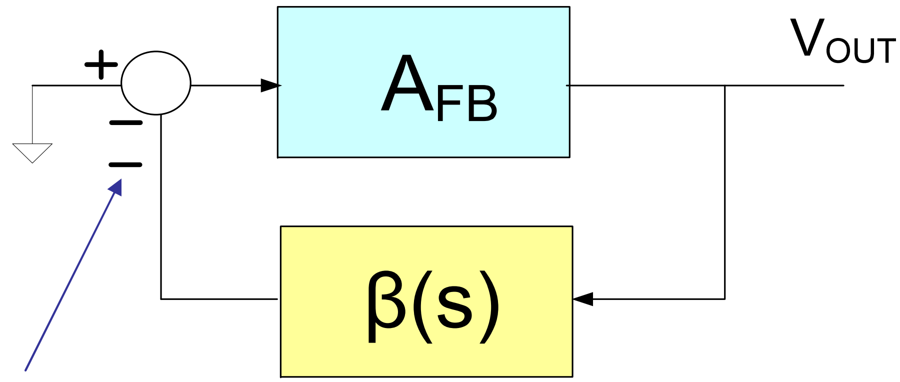
$$R_2 + R_3 > 2R_1$$

$$\omega_{OSC} \cong \frac{1}{RC}$$

Wein – Bridge Oscillator

– an alternative view of same circuit using feedback concepts





Note double inversion

$$\beta(s)=\frac{\left[\frac{R}{1+RCs}\right]}{\left[\frac{R}{1+RCs}\right]+\left[\frac{1+RCs}{Cs}\right]}$$

$$A_{FB}=1+\frac{\left[R_2+R_3\right]}{R_1}$$

$$D(s)=s^2+s\frac{\left[3-A_{FB}\right]}{RC}+\frac{1}{(RC)^2}$$

$$A_{FB}=3+\varepsilon \qquad \omega_{OSC}\cong\frac{1}{RC}$$

$$R_2 < 2R_1$$

$$R_2+R_3 > 2R_1$$

End of Lecture 25